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BURN-OUT OF A LIQUID UNDER CONDITIONS OF NATURAL CONVECTION

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A relation for the critical heat flux density in liquid boiling under conditions of natural convection is derived by means of a previously obtained approximate analytical solution of the problem of the hydrodynamics of an evaporating meniscus.

The classical Kutateladze relation for the critical heat flux density under conditions of natural convection has the form [1, 2]

$$q_* = 0.14r\rho_v^{1/2}(\tilde{g}\sigma\rho)^{1/4}. \tag{1}$$

As is known [3], formula (1) becomes inapplicable for the range of low reduced pressures. In [3] a model of burn-out is developed based on an analysis of the process of evaporation of menisci of a liquid film that lie on boundaries of dry spots under vapor bubbles on the heating surface (Fig. 1). The appearance of burn-out is associated with an increase in the size of the dry spots existing on the heating surface in nucleate boiling.

Using the method of physical estimates of the system of equations describing the hydrodynamics of a meniscus [4-6], the author of [3] obtained relations for the critical heat flux density in the limiting cases of low and high pressures. The resultant interpolation relation constructed in [3] generalizes a large quantity of experimental data on critical heat flux densities in liquid boiling under conditions of natural convection.

In [7, 8] an approximate analytical solution of the system of differential equations [4, 6] describing a flow in an evaporating meniscus is obtained. The correctness of the solution [7, 8] is confirmed by comparing it with a numerical calculation for the case of evaporation of the menicus of liquid ammonia at atmospheric pressure within the range of temperature drops $\Delta T = 1-100$ K. We note that an analysis of the hydrodynamics of an evaporating meniscus on the boundary of a dry spot was used in [10] in determining the surface density of boiling centers.

In the present paper an attempt was made to take the next step in the development of the approach of [3]: to pass from the method of physical estimates in the analysis of burn-out to direct use of the results of solution of the problem of an evaporating meniscus. For this purpose we use the relations of [7, 8] for the density of the heat flux transferred through the meniscus

$$q_m = 1.6\frac{\lambda\Delta T}{\delta_m}\ln\left(0.65\frac{\alpha_k\delta_m}{\lambda}\right), \tag{2}$$

and for the connection between the geometric dimensions of the meniscus (Fig. 1)

$$\frac{\delta_m}{l_m} = 2.1\left(\frac{\alpha_k\Delta T\nu}{\delta r}\right)^{1/4}, \tag{3}$$

where α_k is the kinetic coefficient of heat transfer [11]:

$$\alpha_k = 0.9\rho_v r^{3/2}/T_s. \tag{4}$$

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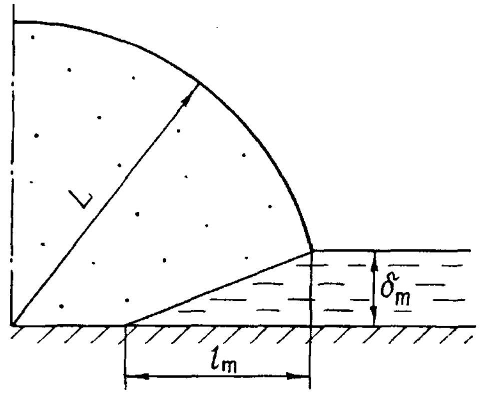

Fig. 1. Schematic diagram of an evaporating meniscus at the boundary with a dry spot under vapor formation on the heating surface in nucleate boiling.

As is known [12, 13], the characteristic size of a vapor bubble on a solid surface as a function of the pressure is determined by either the "inertial scale"

$$L_1 \approx \beta_1 \left(\frac{\lambda \Delta T}{r\rho_v}\right)^{2/3} \frac{1}{\tilde{g}^{1/3}}, \tag{5}$$

or the capillary constant

$$L_2 \approx \beta_2 \sqrt{\left(\frac{\sigma}{\rho \tilde{g}}\right)}. \tag{6}$$

According to [3], a determining role in heat transfer in the precritical region of boiling is played by powerful heat sinks situated at the boundaries of dry spots under vapor formations (Fig. 1). Quantitatively this effect can be taken into account by the interpolation formula

$$\frac{1}{l_m^{n_1}} \approx \frac{1}{L_1^{n_1}} + \frac{1}{L_2^{n_1}}, \tag{7}$$

where the power n_1 is a free numerical constant.

It follows from (7) that the length of the meniscus "is tuned" to the lesser of the two possible characteristic sizes of the vapor bubble: $l_m \to L_1$ when $L_1 \ll L_2$ (the region of high pressures); $l_m \to L_2$ when $L_2 \ll L_1$ (the region of low pressures). In accordance with (2), (3) this means that in the calculation of heat transfer the most intense heat sinks situated at the boundaries of the smallest vapor bubbles are taken into account.

We present the dependence $q_m(\Delta T)$ approximately in the form of a power function $q_m \sim \Delta T^{n_2}$. Replacing the logarithmic function in (2) by a constant, we obtain using (3), (7): $n_2 = 1/12$ for $l_m = L_1$; $n_2 = 3/4$ for $l_m = L_2$.

The dependence $q(\Delta T)$ for nucleate boiling is expressed by the known law [14, 15]

$$q = 10^{-3} \frac{\lambda^2 \Delta T^3}{\nu \sigma T_s}. \tag{8}$$

A comparison of the dependences $q_m(\Delta T)$ and $q(\Delta T)$ gives the following clear physical picture of burn-out. Far from burn-out the process of boiling will be stable due to the fact that the "transmitting capacity" of the meniscus with respect to the heat flux is much larger than the mean value over the surface ($q_m \gg q$). However, with an increase in the temperature drop the value of q will increase much more rapidly than q_m, and eventually the dependences $q(\Delta T)$ and $q_m(\Delta T)$ will intersect (Fig. 2). A hypothetical further increase in ΔT will lead to disruption of the stationarity of heat removal through the meniscus – its "additional feeding" with liquid from the side of the

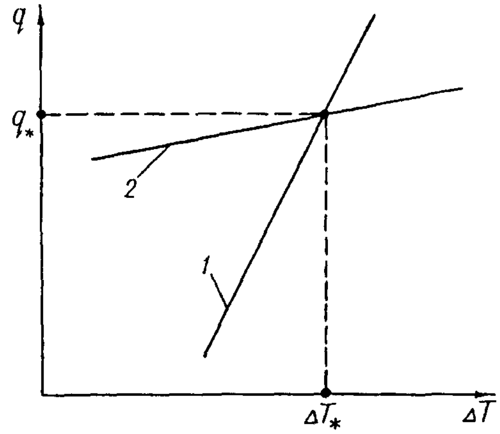

Fig. 2. Dependence of the mean heat flux density over the boiling surface (1) and that removed through the meniscus (2) on the temperature drop.

thick film will be insufficient for compensation of the flow of liquid evaporating from the upper boundary of the meniscus (see Fig. 1). Thus, there should exist some boundary value of the temperature drop ΔT_* that cannot be exceeded under conditions of stable nucleate boiling (Fig. 2). This is the temperature drop to which the critical heat flux density will correspond:

$$q = q_{\mathrm{m}} = q_* . \tag{9}$$

It also follows from the physical model considered that at $\Delta T = \Delta T_*$, $q = q_*$ a nonstationary "subcritical stage" of nucleate boiling where the dry spot under the bubble begins to increase due to motion of the liquid meniscus toward the thick film is realized (Fig. 1). This allows one to determine the characteristic times of surface drying with onset of burn-out, which were studied experimentally in [16].

Substitution of (2), (8) in the condition of burn-out (9) allows, in principle, determination of the value of ΔT_*, and from it the sought value of q_*. The very cumbersome expressions obtained here make it impossible, however, to obtain simple power dependences of the critical heat flux density on the physical parameters, thus impeding a comparison of results of calculation by the present model with known results [1-3].

Therefore, we consider the limiting cases of "high" and "low" pressures, when the logarithmic function in expression (2) for the density of the heat flux transferred through the meniscus can be replaced approximately by power dependences of the form A^{1/n_3}, where $A = 0.65\alpha_{\mathrm{k}}\delta_{\mathrm{m}}/\lambda$. Here we use the approximate thermodynamic relation $c_p T_{\mathrm{s}} \approx r$ [17].

a) In the region of high pressures ($l_m = L_1$; $A \approx 10^4 - 10^6$; $n_3 \approx 7$)

$$q_{*1} \approx k_1 \, \mathrm{Pr}^{-2/13} r^{23/26} \rho_{\mathrm{v}}^{7/13} \rho^{2/13} (\sigma \tilde{g})^{4/13} . \tag{10}$$

b) In the region of low pressures ($l_{\mathrm{m}} = K_2$; $A \approx 10^1 - 10^3$; $n_3 \approx 3$)

$$q_{*2} \approx k_2 \, \mathrm{Pr}^{-2/13} r^{19/26} \rho_{\mathrm{v}}^{3/13} \rho^{8/13} \sigma^{2/13} \tilde{g}^{6/13} \nu^{4/13} . \tag{11}$$

It follows from (10), (11) that the values of q_* obtained for the two limiting cases are related to each other as

$$q_{*2}/q_{*1} \approx \left(\frac{\nu^2 \tilde{g} \rho^3}{r \rho_{\mathrm{v}}^2 \sigma} \right)^{2/13} . \tag{12}$$

We compare the obtained relations with the results of [1-3] by introducing the "relative" critical heat flux density ψ – the value of q_* determined by relations (10), (11) of the present paper is inserted into the numerator, and the corresponding "reference" value of q_* from [1-3] is inserted into the denominator.

a) In the region of high pressures ($\psi = \psi_1$), the reference relation is the formula [1, 2]

$$\psi_1 = \frac{\rho_v^{1/26} (\sigma \tilde{g})^{3/52}}{\rho^{5/52} r^{3/26}} . \quad (13)$$

Here, due to the very low powers of all the parameters entering (13) it is evident that formula (10) virtually coincides with classical relation (1). This makes it possible to determine the free numerical constant $k_1 \approx 2.4$.

b) In the region of low pressures ($\psi = \psi_2$), the reference relation is the formula [3]

$$\psi_2 = \frac{\rho_v^{1/455} r^{93/910} \rho^{254/455} \tilde{g}^{54/455} \nu^{309/455}}{\mathrm{Pr}^{153/910} \sigma^{51/91}} \quad (14)$$

Here the powers of the vapor density and the heat of phase conversion virtually coincide. This indicates that formula (11) correctly reflects the characteristics of low pressures. However, the difference in the powers of the kinematic viscosity, surface tension, and Prandtl number is not small. This makes it impossible to determine the free numerical constant k_2. Thus, the applicability of the present model to the region of very low pressures is limited.

We note that the resultant interpolation relation [3] has the following structure:

$$q_*^{n_4} = q_{*1}^{n_4} + q_{*2}^{n_4} . \quad (15)$$

Here q_{*1}, q_{*2} are the corresponding limiting values for high and low pressures, each of which involves a free numerical constant; $n_4 = 5/2$ is the power selected from a comparison with a large quantity of experimental data. The present approach contains, in principle, three free numerical constants β_1, β_2, and n_1 in formulas (5)-(7). We note that the constants k_1, k_2 in formulas (10), (11) that appear after power approximation of the logarithmic function in (2) are unique functions of the "primary constants" β_1, β_2.

Since the correctness of relation (10) is confirmed by its good agreement with the Kutateladze formula (1), this makes it possible to evaluate the rate of surface drying with onset of burn-out for the region of high pressures by means of the heat balance of the evaporating meniscus

$$u_m = \frac{q_{*1}}{r\rho} \frac{l_m}{\delta_m} , \quad (16)$$

and relations (3), (10).

NOTATION

ρ_v, vapor density; ρ, liquid density; ν, kinematic viscosity of the liquid; λ, thermal conductivity of the liquid; c_p, specific heat of the liquid; r, heat of phase conversion; σ, coefficient of surface tension; δ_m, l_m, thickness and length of the meniscus; ΔT, temperature drop; q, heat flux density; q_*, critical heat flux density; q_{*1}, q_{*2}, the same for the regions of high and low pressures; T_s, saturation temperature; g, free-fall acceleration; $\tilde{g} = g(\rho - \rho_v)/\rho$; Pr, Prandtl number for the liquid; L_1, L_2, characteristic dimensions of the vapor bubble for the regions of high and low pressures; ψ, relative critical heat flux density; u_m, rate of surface drying; β_1, β_2, k_1, k_2, numerical constants; n_1, n_2, n_3, n_4, powers.

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